Title: The Importance of The Place of Defuzzification Step In Fuzzy Systems

Authors: Yakup Çelikbilek
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The Importance of the Place of Defuzzification Step In Fuzzy Multi Criteria Decision Making

Yakup Çelikbilek

Abstract

Fuzzy logic and fuzzy based techniques are applied constantly in both social sciences and engineering sciences for fuzzy systems. However, theoretical content is neglected occasionally in these developed methods. Even sometimes, propositions and applications are developed by using fuzzy numbers ignoring theoretical contents. In some cases, there are lots of different fuzzy solution propositions of same crisp method. Because of these, despite applying similar methods to same data set, very different results can be obtained. Fuzzy multi criteria decision making method propositions are evaluated by using simulation technique in this study to explore and compare the results of fuzzy applications. Simulation applications are applied to AHP, TOPSIS, VIKOR and MOORA methods which are the most common multi criteria decision making methods as both crisp and fuzzy in the literature. Obtained results are evaluated and interpreted for both selection of the best alternative and ranking of all alternatives. All obtained results of simulation applications show that defuzzified results are significantly different than each other for each multi criteria decision making method and each different defuzzification step.

Keywords: Fuzzy Systems, Fuzzy Numbers, Defuzzification, Multi Criteria Decision Making, Simulation.

1. INTRODUCTION

Fuzzy based applications have been commonly used at both academic studies and industrial applications since it was introduced. Approaches developed by using fuzzy logic (FL), especially in social sciences, is used to decrease subjective judgements during the solutions of problems. To ensure usage of FL especially in fuzzy systems (FS), fuzzy based applications and proposals have been developed increasingly day by day. But, most of the new fuzzy applications and proposals does not consider the previous approaches and theoretical contents. They primarily focused on the integration and application of FL with the other methods. Because of these, there are lots of fuzzy based methods overlooked the main theoretical contents in the literature. Defective applications and proposals have been also diffused increasingly with their use in other studies. These problems have cause misdirection of FL based applications and FS literature. Methods intended to decision making results with defective decisions. The main difference of the applications for FS especially in decision making problems is the place of the defuzzification step. The difference is mostly observed in the applications of social sciences. In some studies, fuzzy data set obtained by using linguistic scales for FS applications in multi criteria decision making (MCDM) problems is defuzzified at the beginning of the solution. In some other studies, fuzzy data set is defuzzified right
after the generation of decision matrix. Beside these, in the few rest of the other studies, fuzzy data set is defuzzified at the end of the study after obtaining the solution of the problem according to the FL and FS rules.

All of the fuzzy based approaches uses the theoretical logic of the crisp methods integrated. Results of the studies seem close to each other but basically all of them produces different results than each other. These differences of the results based on theoretical basis of the studies or the differences of numerical calculations of fuzzy numbers (FN). In this study, simulation applications based on the defuzzification in different steps of different methods are carried out to explore the importance of the place of defuzzification step. To decrease the differences because of theoretical content or numerical calculations, the fuzzy method proposals, which are explained in detail in the following sections, based on the same foundations are chosen. The importance of the place of defuzzification in FS is analyzed and evaluated by detailed comparative tables. Obtained results of the study show that the place of defuzzification step is important and effective on the actual results of the problems.

In line with these objectives, in the second section, applied fuzzy multi criteria decision making (FMCDM) methods are evaluated with the applications and studies in the literature. The third section is allocated to simulation applications and their results. Obtained simulation results are evaluated and interpreted in detail in the fourth section. In the last section as discussion and conclusion, the study is resulted with a general evaluation and recommendations for future studies.

2. FUZZY MULTI CRITERIA DECISION MAKING

Almost all of the proposed MCDM problems in the literature has a fuzzy version. In some studies, all of the calculations are carried out with FN and FS rules while some other studies defuzzify the fuzzy decision matrix before starting the calculations and continue the solution by using crisp MCDM methods. Numbers of proposed fuzzy methods and their variants in the literature depend on the easiness of the implementation of MCDM method with FS under fuzzy rules. For instance AHP, it is easy to integrate and implement TOPSIS or MOORA methods with FS and its calculations. Because of these, there is not many approaches for fuzzy TOPSIS and fuzzy MOORA. But as well, all of the proposed approaches for fuzzy TOPSIS and fuzzy MOORA are different than each other at least in one step. In the following sub-sections, four most common MCDM methods also used in this study are given with detailed explanations and literature instances under the related sub-sections.

AHP, TOPSIS, VIKOR and MOORA methods are chosen for the applications in this study. The reason for choosing these four methods in the applications is the excess of the literature about these methods in both crisp systems and FS. Fuzzy based literature for the other MCDM methods are not common as these four methods.

2.1. Fuzzy AHP

AHP (Analytic Hierarchy Process) was developed and introduced first time in 1980 by Saaty [1]. The method is based on pairwise comparisons of alternatives according to criteria. It is preferred commonly for MCDM problems as both selection and ranking in the literature. It is also preferred commonly for FS solutions of MCDM problems as fuzzy AHP method. There is a wide variety of fuzzy based AHP propositions in the literature. Fuzzy AHP propositions in the literature mainly defuzzify FS in two different steps of the propositions as the other MCDM methods in the literature. Some propositions in the literature [2–8], which prefer defuzzification at the beginning of the solution right after the generation of decision matrix, use FN just for the pairwise comparisons. After obtaining the defuzzified decision matrix as crisp decision matrix, they continue the rest of the solution with the crisp AHP method. The other fuzzy AHP propositions in the literature [9–12] complete all of the calculations with FN until the end of solution. They obtained all of the result including priority vector with FN, and evaluate and compare the alternatives according to the results with FN. Defuzzifications in these studies are done at the end of the fuzzy AHP proposition.

Fuzzy AHP simulations in this study is carried out with fuzzy AHP method proposed by Çelikbilek et al. [12]. The reason of chosen the proposed fuzzy AHP method is that calculations of the method is conducted with FN from the beginning to the end of the proposition. This approach provides convenience for the simulations and
comparison of defuzzification in different steps. Because, defuzzifications are done in different steps of the proposed fuzzy AHP method to observe the importance and progress of continuing the calculations with FN. Details of the steps of the fuzzy AHP can be found in [12].

2.2. Fuzzy TOPSIS

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method was firstly introduced by Hwang and Yoon [13] in 1981. Theoretical content of the method is based on the idea of distance among alternatives, non-ideal solution and ideal solution. If an alternative is far to the non-ideal solution, it is close to the ideal solution at the same time. The method uses the Euclidean distance for the distance calculations. Therefore, it is more difficult to develop a fuzzy approach for TOPSIS method as compared to the others. Because of this, while there are various fuzzy TOPSIS propositions in the literature, there is not many propositions for fuzzy TOPSIS as fuzzy AHP. Like fuzzy AHP propositions, some propositions in the literature [14–20] prefers defuzzification at the beginning of the solutions to generate crisp decision matrix for crisp TOPSIS calculations. The other fuzzy TOPSIS propositions in the literature [21–23] complete all calculations with FN until the end of solutions. Defuzzifications in these studies are done at the end of fuzzy TOPSIS propositions from the fuzzy weights of alternatives.

Fuzzy TOPSIS simulations in this study is carried out with fuzzy TOPSIS method proposed by Büyüközkân and Güleryüz [23]. Like the reason in fuzzy AHP proposition, calculations of the proposed fuzzy TOPSIS is conducted with FN from the beginning to the end of the proposition. Details of the steps of the fuzzy TOPSIS can be found in [23].

2.3. Fuzzy VIKOR

VIKOR (Vlse Kriterijumska Optimizacija I Kompromisno Resenje) method was introduced by Opricovic [24] in 1998. Regret criterion in decision theory form the basis of the method. Although not as many as fuzzy AHP propositions, many researchers, especially Opricovic [25], have developed various fuzzy VIKOR propositions. Like fuzzy AHP and fuzzy TOPSIS propositions, some fuzzy VIKOR propositions in the literature [26–32] prefer defuzzification at the beginning of the solution to generate crisp decision matrix for crisp VIKOR calculations. The other fuzzy VIKOR propositions in the literature [25, 33–38] complete all of the calculations with FN until the end of solution and defuzzifications in these studies are done at the end of the fuzzy VIKOR proposition from the fuzzy weights of alternatives.

Fuzzy VIKOR simulations in this study is carried out with fuzzy VIKOR method proposed by Çelikbilek and Tüysüz [38]. Like the reason in fuzzy AHP proposition, calculations of the proposed fuzzy VIKOR is conducted with FN from the beginning to the end of the proposition. Details of the steps of the fuzzy VIKOR can be found in [38].

2.4. Fuzzy MOORA

MOORA (Multi-Objective Optimization on basis of Ratio Analysis) method was introduced by Brauers ve Zavadskas [39] in 2006. Although the basis of the method is similar with TOPSIS, MOORA method conduct the calculations only with the ideal solution. Through the calculations without using the Euclidean distance unlike in TOPSIS method, it is easier to develop a fuzzy approach for MOORA method relative to TOPSIS. As one of the newest MCDM method, fuzzy MOORA propositions and their applications are not as many as the other MCDM methods. Like the other FMCDM propositions, some fuzzy MOORA propositions in the literature [40–42] prefer defuzzification at the beginning of the solution to generate crisp decision matrix for crisp MOORA calculations. The other fuzzy MOORA propositions in the literature [9, 43–45] complete all of the calculations with FN until the end of solution and defuzzifications in these studies are done at the end of the fuzzy MOORA proposition from the fuzzy weights of alternatives.

Fuzzy MOORA simulations in this study is carried out with fuzzy MOORA method proposed by Karande and Chakraborty [45]. Like the reason in fuzzy AHP proposition, calculations of the proposed fuzzy MOORA is conducted with FN from the beginning to the end of the proposition. Details of the steps of the fuzzy MOORA can be found in [45].
3. SIMULATION

The details of simulation applications to analyze the results of selecting the place of defuzzification step in different parts are given in this section. The main aim of simulations is to observe the changes of selecting the best or the worst alternatives and ranking of all alternatives according to defuzzification in different steps.

Simulation applications are carried out by using random data with 20 alternatives and 10 criteria. Table 1 proposed by [12] is used for generating and the evaluations of random data. An example of random fuzzy decision matrices (9^200 matrix combinations) by using Table 1 is given in Table 2. In each simulation, random fuzzy decision matrices are chosen from 9^200 matrix combinations where each combination includes different random fuzzy numbers.

### Table 1: Linguistic fuzzy scales [12].

<table>
<thead>
<tr>
<th>Crisp Value</th>
<th>Linguistic term</th>
<th>Fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally Important (EI)</td>
<td>(1,1,2)</td>
</tr>
<tr>
<td>3</td>
<td>Weakly Important (WI)</td>
<td>(2,3,4)</td>
</tr>
<tr>
<td>5</td>
<td>Important (I)</td>
<td>(4,5,6)</td>
</tr>
<tr>
<td>7</td>
<td>Strongly Important (SI)</td>
<td>(6,7,8)</td>
</tr>
<tr>
<td>9</td>
<td>Absolutely Important (AI)</td>
<td>(8,9,9)</td>
</tr>
</tbody>
</table>

After generating random data, evaluations of random data sets are carried out by using four FMCDM methods introduced in the second section. All of the simulation applications of four FMCDM methods is done by using same random data sets also for comparison among the proposed methods. Converting Fuzzy data into Crisp Scores introduced by Opricovic and Tzeng [46] is used for the defuzzifications in all simulation applications. The aim of using the same defuzzification method instead of the proposed defuzzification methods in the articles referred in the second section is to control and compare the methods with less subjectivity.

Defuzzifications in simulation applications are done in two different steps according to literature review of FMCDM methods given in the previous section. First defuzzification (S2) is done after the generation of decision matrix and before starting the calculations of MCDM methods. This means that calculations are done by using crisp MCDM method rules instead of fuzzy. The other defuzzification (S3) is done at the end of FMCDM methods after obtaining the final results with FN. Furthermore, generated random data sets are also analyzed by using crisp number representations given in Table 1 with crisp MCDM methods (S1) to evaluate and compare reciprocallly.
Defuzzification versions done in different steps of simulation applications are given in detail below.

**S1:** Simulations are carried out by using crisp numbers with crisp MCDM methods.

**S2:** Defuzzifications are done right after the generation of decision matrix and also before starting the calculations of MCDM methods. In this version after the defuzzification, all of the calculations are done by using crisp MCDM method rules [1, 13, 24, 39] instead of fuzzy.

**S3:** Defuzzifications are done at the end of the FMCDM methods after obtaining the final results with FN. In this version, all of the calculations are done by using FMCDM method rules without crisp numbers or crisp rules.

These three different simulation application versions are simulated 10,000 times with the same random data sets in each run of simulation applications for all MCDM methods. To this aim and because of the various connected matrix calculations in each method, MS Office Excel software is used to obtain all of these results at the same time. MS Office Excel Formulas are used in the simulation applications according to the theories of MCDM and FMCDM methods.

The details of the sections and the tables are also given in detail in the related section.

### 3.1. Selecting the best and the worst alternatives

Most of the studies with MCDM methods is to select the best alternative. While selecting the best alternative by using a MCDM method, it is also important to determine the worst alternative correctly as well as selecting the best alternative. Because of these, comparison results of selecting the best and the worst alternatives are given in this section in detail.

Dual combination results of simulation applications for the best alternatives are given in Table 3. In the second and third column of Table 3, dual combination results of 10,000 simulations are given with the percentages where the same best alternative is obtained. In the fourth and fifth column of Table 3, dual combination results of 10,000 simulations are given with the percentages where the best alternative is obtained in the first or second of the ranking list reciprocally.

<table>
<thead>
<tr>
<th>Comparisons</th>
<th>S1–S3</th>
<th>S2–S3</th>
<th>S1–S3</th>
<th>S2–S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy AHP</td>
<td>54.3</td>
<td>74.9</td>
<td>76.9</td>
<td>93.7</td>
</tr>
<tr>
<td>Fuzzy TOPSIS</td>
<td>53.5</td>
<td>56.9</td>
<td>75.9</td>
<td>80.4</td>
</tr>
<tr>
<td>Fuzzy VIKOR</td>
<td>63.6</td>
<td>67.2</td>
<td>85.6</td>
<td>88.8</td>
</tr>
<tr>
<td>Fuzzy MOORA</td>
<td>57.8</td>
<td>65.1</td>
<td>77.8</td>
<td>91.5</td>
</tr>
</tbody>
</table>

Dual combination results of simulation applications for the worst alternatives are given in Table 4. In the second and third column of Table 4, dual combination results of 10,000 simulations are given with the percentages where the same worst alternative is obtained. In the fourth and fifth column of Table 4, dual combination results of 10,000 simulations are given with the percentages where the worst alternative is obtained in the first or second of the ranking list reciprocally.

<table>
<thead>
<tr>
<th>Comparisons</th>
<th>S1–S3</th>
<th>S2–S3</th>
<th>S1–S3</th>
<th>S2–S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy AHP</td>
<td>58.2</td>
<td>65.1</td>
<td>81.2</td>
<td>84.7</td>
</tr>
<tr>
<td>Fuzzy TOPSIS</td>
<td>60.1</td>
<td>65.5</td>
<td>87.6</td>
<td>89.9</td>
</tr>
<tr>
<td>Fuzzy VIKOR</td>
<td>66.8</td>
<td>77.5</td>
<td>91.4</td>
<td>95.1</td>
</tr>
<tr>
<td>Fuzzy MOORA</td>
<td>61.0</td>
<td>68.7</td>
<td>85.1</td>
<td>94.5</td>
</tr>
</tbody>
</table>

Obtained simulations results in Table 3 and Table 4 show that approximately 50 % of the best alternative results (S1–S3) are different than the actual the best result of the FS if a FS is evaluated by using crisp MCDM methods instead of fuzzy version. For instance in Table 3, 54.3 % of the best alternatives selected by using fuzzy AHP method is also the best alternatives selected by using crisp AHP method. However, this also means that 45.7 % of the best alternatives selected by using fuzzy AHP method is not selected as the best alternatives by using crisp AHP method. Similarly in
Table 3, for the other methods as TOPSIS, VIKOR and MOORA, 53.5 %, 63.6 % and 57.8 % of the best alternatives selected by using fuzzy methods are the best alternatives selected by using crisp methods respectively. Likewise, this means that 46.5 %, 36.4 % and 43.2 % of the best alternatives selected by using fuzzy methods are not selected as the best alternatives by using crisp methods respectively.

Similarly, in the fourth column of Table 3, 76.9 % of the best alternatives selected by using fuzzy AHP method is selected as the best alternatives or the second alternatives by using crisp AHP method. With the difference between the second column and the fourth column, this means that 22.6 % of the best alternatives of FS is selected as the second best alternative by using crisp AHP. All of the table can be read as this.

The differences between the second column and the third column show the increases if the FN are kept until generating the fuzzy decision matrices. These differences in Table 3 and Table 4 are about 10 % in average. This 10 % can be interpreted as the increase ratio of correct decisions if fuzziness is kept more in the system. All of these results in Table 3 and Table 4 shows that the accuracy ratios of FS increases gradually depend on how long the fuzziness is kept in the system until obtaining the final results with FN.

T-tests were also carried out to determine the statistical significance among the dual combinations of each FMCDM method. According to the test results, most of the significance level results is lower than 0.001 (P<0.001), and the others are lower than 0.005 (P<0.005). These results demonstrate that all null hypotheses are rejected for conventional levels of statistical significance. By rejecting the null hypotheses, interpretations about Table 3 and Table 4 above is verified.

### 3.2. Ranking of all alternatives

In this sub–section, comparison results of ranking all alternatives are given in detail. In the second and third columns of Table 5, dual combination results of 10,000 simulations are given with the percentages where the same ranking is obtained for each alternative reciprocally. In the fourth and fifth columns of Table 5, dual combination results of 10,000 simulations are given with the percentages where the ranking is obtained one before or one after the ranking of the other simulation version ranking list for each alternative reciprocally.

**Table 5: Comparison results of ranking all alternatives.**

<table>
<thead>
<tr>
<th>Comparisons</th>
<th>The same ranking (%)</th>
<th>The one before or after (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1–S3</td>
<td>30.0</td>
<td>43.3</td>
</tr>
<tr>
<td>S2–S3</td>
<td>31.0</td>
<td>34.9</td>
</tr>
<tr>
<td>Fuzzy AHP</td>
<td>30.0</td>
<td>43.3</td>
</tr>
<tr>
<td>Fuzzy TOPSIS</td>
<td>31.0</td>
<td>34.9</td>
</tr>
<tr>
<td>Fuzzy VIKOR</td>
<td>39.3</td>
<td>43.2</td>
</tr>
<tr>
<td>Fuzzy MOORA</td>
<td>33.6</td>
<td>37.7</td>
</tr>
<tr>
<td></td>
<td>64.7</td>
<td>64.6</td>
</tr>
<tr>
<td></td>
<td>79.5</td>
<td>69.5</td>
</tr>
<tr>
<td></td>
<td>81.2</td>
<td>81.2</td>
</tr>
</tbody>
</table>

There are serious declines of the comparison results of ranking all alternatives given in Table 5 relative to Table 3 and Table 4. Especially the second and the third columns of the table decline about half of the ratios in Table 3 and Table 4. These results show that 70.0 %, 69.0 %, 60.7 % and 66.4 % of the alternatives are ranked in incorrect rankings according to FS rankings of the methods in Table 5 respectively. The differences between the second column and the fourth column are about 30 % in average. This 30 % means the ratio of 1 approximate results between the rankings obtained by using crisp methods and the rankings according to fuzzy methods. Even according to the fifth column, which has the highest ratios with 1 approximate results, 20.5 %, 30.5 %, 18.8 % and 31.9 % of the alternatives are ranked in incorrect rankings according to FS rankings of the methods in Table 5 respectively. These incorrect rankings are more than 2 approximate rankings according to FS rankings. If we consider these with high ratios given in Table 3, Table 4 and Table 5, FS should be considered as much as possible with FN instead of crisp numbers to obtain much better results for decisions.

Same as the previous sub-section, t-tests were carried out to determine the statistical significance among the dual combinations of each FMCDM method. According to the test results, all of the significance level results is lower than 0.001 (P<0.001). These results demonstrate that all null hypotheses are rejected for conventional levels of statistical significance. By rejecting the null hypotheses, interpretations about Table 3 and Table 4 above is verified.
4. INTERPRETATION OF THE RESULTS

All obtained results of simulation applications show that defuzzified results are significantly different than each other for each MCDM method. Especially because of the structures, evaluating FS by defuzzifying them both restrict the results and cause incorrect decision making. In other words, defuzzified values are final values and restricted than the fuzzy values. However, fuzzy values are not restricted and include all of the possible values of the FS. In this way, all of the possible values can be included in the final solution of the problem. Therewithal, alternatives can be also evaluated with their similarities observed by using intersections of fuzzy values. An alternative can be resulted as the best alternative with defuzzified values but it can be intersected more than 95% with the fuzzy value of second alternative. Even in this kind of cases, second alternative can be resulted as the first alternative depends on the defuzzification technique used in the proposed method. Because of these, in a FS, keeping out of evaluation of all alternatives excluding the best alternative with defuzzified values can cause significant incorrect decisions. Differences between exact results and 1 approximate results in Table 3, Table 4 and Table 5 are between 20% and 35% which also mean the ratio of the first alternative taking place of the second alternative by defuzzifying at the beginning of the systems. This is the ratio of the best alternative which take place of the second alternative, not the rest of the rankings. If we consider the best alternatives which take place of the rest of the rankings, the ratio of incorrect decisions increases much more. These also can be observed with complementary ratios in Table 3, Table 4 and Table 5. All complementary ratios of Table 3, Table 4 and Table 5 mean the ratio of incorrect decisions taken by using crisp values defuzzified at the beginning than fuzzy values.

These significant results and ratios are especially much more important for the decisions such as facility location selection, industrial equipment selection, investment sector selection or governmental decisions, etc. with high budgets. These kind of decisions should be evaluated in more detail especially in sensitive FS. Evaluations of this kind of sensitive FS can be carried through easily with fuzzy values, while it cannot be carried through easily with defuzzified, crisp values of FS because of the restricted results with unseen bounds. The unpredictable results of this kind of restricted calculations of FS can be resulted with bankruptcies of companies.

CONCLUSION AND DISCUSSIONS

The importance and the effects of the place of defuzzification step in FS are analyzed by using four different MCDM methods which are AHP, TOPSIS, VIKOR and MOORA. For fuzzy approaches of each MCDM methods used in the study, the most descriptive, smooth methods which obtain results with FN without any problems in the literature are chosen. Therewith, after defuzzification in different steps of the proposed methods, calculations can be continued with crisp versions of MCDM methods. And, the results of defuzzification in different steps of same fuzzy approaches are also obtained in detail without subjectivity. The results of this study show that the correctness of the evaluation results is directly proportional to the closeness of the defuzzification step to the end of the methods. In this case also therefore, defuzzified results eliminate the possibility of evaluation and interpretation of the results flexibly. If many various defuzzification techniques in the literature are also considered, restriction of controlling possible results at the beginning of evaluations in FS produce incorrect results and make impossible of observing the origin of faults. Obtaining all results with FN provides opportunity to compare the alternatives and the possible results of FS through wider perspectives. At the same time, bounds of FS components can be observed clearly to act according to them.

The other important point of this study is providing opportunity to review and making perfect existing FMCDM methods which was proposed with ideas of theories of classical system and MCDM methods by using the results and reviews in this study.

For further studies, the results and reviews of this study can be expanded through other FS theories to propose better methods for FS. Omissions of existing methods can be detected to improve them. Besides all these, unsolved problems of FS can be reviewed and solutions can be researched again through the results obtained in this study, existing theories and methods.
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