Vehicle Dynamics Simulations on Road and Off-Road Surfaces – A Comparative Analysis

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ABSTRACT

Tyre models used in vehicle dynamics models are generally developed based on the behaviour of vehicles on specially constructed roads with prescribed adhesion characteristics. The behaviour of road and off-road vehicles on two different surfaces is different. In this study; on the basis of a tyre model that can be adapted both on the road and off-road, an approach is shown to the actual or encountered situations with simulated comparisons over standard vehicle dynamics manoeuvres. Matlab / Simulink is used as modelling and simulation environment.

Keywords: off-road tyre model, vehicle dynamics, terrain vehicles, handling dynamics

1. INTRODUCTION

Vehicle Dynamics studies need a proper representation of the interaction between the tyre and road surface. There are some well-known tyre models like Pacejka’s Magic formula [1] in the literature; those are highly reliable in vehicle dynamics simulations to represent the behaviour of the tyre on paved surfaces, for both academic and commercial purposes. However, behaviour of an off-road vehicle on the pavement is just a fraction of its entire service life. The tyre models developed for pavements focus on the construction due to shear mechanisms of adhesion and tyre-ground sliding. During the off-road drive, on the other hand, tyre-terrain interaction and soil deformation is more important [2]. Therefore, it is important that to reveal the dominant part by using a tyre model which can extend the force approximation capabilities for unpaved surfaces.

Bekker and Wong [3, 4] have been dominated this area with their research, especially, since 1960s. Bekker developed the bevameter technique which is based on measuring the terrain properties under loading conditions similar to those exerted by an off-road vehicle [5]. He considers wheels as simple loading surfaces having similar forms but different lengths and widths [6]. Especially, under soft terrain conditions, tyres penetrate the surface due to the individual tyre load and the amount of shear displacement [7]. As a result of these, rolling resistance also increases. The surface quality is on the other hand, another fact that determines the coefficient of friction as it is occurs for paved surfaces. According to Bekker [6], the maximum shearing force is not developed instantaneously, during the initial part of the motion. It is satisfied after the compaction of soil to some degree. Therefore, some amount of slipping before reaching the maximum traction is inevitable.

When it is compared to numerous researches on tractive capabilities of off-road vehicles, little effort has been made on handling behavior [8]. Metz [9] has developed a set of lateral force equations for off-road surfaces and provided coefficients for several soil conditions.

In this study, a comprehensive modified tyre base model representing both road and off-road capabilities by using the Metz’s equations, has

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been used with an eight-degree of freedom vehicle model to reveal the comparative handling behaviours of a terrain vehicle on road and off-road. The model includes both the bulldozing force and the compaction resistance in the representation of lateral force component.

2. TYRE MODEL

Tyre model used in this study is based on the work of Allen et al. [1, 10, 11]. Although, the model was originally developed for the paved surfaces, it has been further improved by implementing the capability of modelling off-road [2, 7]. The base model takes into account the experimentally provided tyre data to derive analytical and nonlinear solutions to the vehicle dynamics simulations. The model presents combined lateral and longitudinal tyre forces at the same time. These forces can be derived in a load normalised form, by using the equations 1 and 2 by taking the longitudinal ($S$) and lateral slips ($\alpha$) as parameters.

$$\frac{F_y}{\mu F_z} = \frac{f(\alpha)K_s\tan \alpha}{\sqrt{K_i^2\tan^2 \alpha + K_i'^2 S^2}} + Y_f \gamma$$

$$\frac{F_z}{\mu F_z} = \frac{f(\alpha)K_i'S}{\sqrt{K_i^2\tan^2 \alpha + K_i'^2 S^2}}$$

In the equations; $f(\sigma)$ refers to force saturation function which can be determined by composite slip ($\sigma$). Composite slip accounts for both lateral ($K_s$) and longitudinal slip stiffness coefficients ($K_i$) and changes in tyre contact patch length ($a_p$).

$$\sigma = \frac{\pi a_p^2}{8\mu F_z} \left( K_i^2 \tan^2 \alpha + K_i'^2 \left( \frac{S}{1-S} \right)^2 \right)$$

Equation 3 makes possible to define the normalised composite force as shown in the equation 4. In the equation, $C1, C2, C3$ and $C4$ are Calspan coefficients and experimentally determined specifically for a tyre.

$$f(\sigma) = \frac{F_z}{\mu F_z} = \frac{C_1 \sigma^3 + C_2 \sigma^2 + (4/\pi)\sigma}{C_1 \sigma^3 + C_2 \sigma^2 + C_4 \sigma + 1}$$

2.1. Off-Road Capability

Force saturation function includes two polynomials. The roots of the polynomials can be located to present the variation in the shape of the saturation function to accommodate paved and off-road surfaces [9]. Allen’s tyre model uses some shaping functions for the addition of off-road capability. For the reason, Metz’s [9] empirical off-road tyre lateral force model was implemented to the base model by determining the saturation function shaping parameters to match the exponential shape of Metz’s model. To implement the Metz’s model the longitudinal slip is set $S=\gamma=0$ which provides the reduced equations:

$$\frac{F_y}{F_z} = -\mu_f, f(\sigma)$$

$$\sigma = \frac{\pi a_p^2}{8F_z} \frac{K_i \tan \alpha}{\mu_{p_y}}$$

As it is already mentioned, Metz’s model is based on exponential function of tyre slip angle with the parameter relating to cornering stiffness being a function of vertical load [7]. This exponential model is given in the equations 7 and 8, below.

$$\frac{F_{ss}}{F_z} = A(1-e^{-B\alpha})$$

$$B = \frac{C \left( \frac{F_{sz}}{F_z} \right)^n + D}{A}$$

The exponential form of $F_{ss}$ (shear stress to soil deformation) has been shown to fit a variety of tyres and soil conditions [2]. In this equation, $A$ is the equivalent maximum lateral force; $B$ is the cornering stiffness corresponding to the $A$.

Pressure sinkage is also an important parameter in off-road surface and tyre interaction. For homogenous soils, Bekker [4, 5, 6] developed a pressure sinkage relation supported by bevameter technique. If the inflation pressure of a tyre is sufficiently high and there is a relatively soft terrain, then, the deformation of the tyre would be insignificant and it can behave like a rigid tyre. For the case, motion resistance $R$, supporting force of the terrain $W$ and normal pressure beneath the terrain, $p$, at the depth $z$ are given with the equations 9 - 11, where, $n, k_c$, and $k_s$ are pressure-sinkage parameters and $b_r$ is the width of the tyre.

$$p = \left( \frac{k_c}{b_r + k_s} \right)z^n$$

$$R = b_r \int pdz$$

$$W = b_r \int pdx$$

For the steady state conditions, terrain supporting force is equal to the tyre vertical load. Therefore, tyre sinkage can be given by the equation 12.
However, at high slip angles, tyre sinkage develops additional lateral force at the tyre sidewall called “bulldozing force”, \( F_{ys} \), which provides additional lateral force and increases cornering stiffness unlike the soil shear behaviour. As a result, total lateral force is, actually, the sum of the force due to shear stress and bulldozing effect as it is given in Equation 13.

\[
F_y = F_{ys} + F_{yb}
\]  

(13)

Further details about the model can be found in the studies of Allen et al., in the literature [2, 7, 9, 12].

### 3. VEHICLE MODEL

An eight-degree of freedom vehicle dynamics model is used in this study (Equations 14 - 19). In the equations; \( F_x \) and \( F_y \) are the longitudinal and lateral forces, \( M_x \) and \( M_y \) are the moments around longitudinal and lateral axis, \( m \) is the vehicle mass, \( U \) and \( V \) are longitudinal velocities while dot products refer to accelerations, \( I_{xx} \) and \( I_{yy} \) are inertial moments of vehicle mass around \( x \) and \( y \) axes, and \( \dot{p}, \dot{\psi}, \dot{\dot{\psi}} \) are roll acceleration, yaw rate and yaw acceleration, respectively. The model takes into account lateral, yaw, longitudinal and roll motions, enabling the inclusion of traction and braking forces on handling manoeuvres and additionally, the dynamics of each wheel. However, the analysis aimed for this first part of the study is limited, especially to the lateral behaviour of the vehicle with constant longitudinal speed.

\[
\sum F_x = m(\dot{U} + \psi \dot{V})
\]  

(14)

\[
\sum F_y = m(\dot{V} + \psi \dot{U})
\]  

(15)

\[
\sum M_x = I_{xx} \dot{p}
\]  

(16)

\[
\sum M_z = I_{zz} \dot{\psi}
\]  

(17)

Rotational acceleration \( (\dot{\omega}) \), tyre slip angle \( (\alpha) \) and vertical load distribution for each front \( (f) \) and rear \( (r) \) wheels \( (F_z) \), are given in the equations from 18 to 23, respectively. In the equations, \( (\delta) \) is tyre steer angle, \( (\alpha) \) is acceleration, \( (l) \) is vehicle track and \( (L) \) is wheelbase of the vehicle.

\[
\dot{\omega}_{f(l,x)} = \frac{1}{I_{\omega}} \left( - F_{g(f,l,x)} R + M_{a(f,l,x)} \right)
\]  

(18)

\[
\dot{\omega}_{r(l,x)} = \frac{1}{I_{\omega}} \left( - F_{g(r,l,x)} R + M_{a(r,l,x)} \right)
\]  

(19)

\[
\alpha_{f(l,x)} = \delta_{f(l,x)} - \tan^{-1} \left( \frac{V + a \dot{\psi}}{U + \frac{t_r}{2} \dot{\psi}} \right)
\]  

(20)

\[
\alpha_{r(l,x)} = \tan^{-1} \left( \frac{V - b \dot{\psi}}{U + \frac{t_r}{2} \dot{\psi}} \right)
\]  

(21)

\[
F_{zf(l,x)} = \frac{m_f g}{2} - \frac{a_m h}{2t_r} - \frac{a_m h}{2L} \frac{t_r}{t_r} \pm M_{g}
\]  

(22)

\[
F_{zr(l,x)} = \frac{m_f g}{2} + \frac{a_m h}{2t_r} + \frac{a_m h}{2L} \frac{t_r}{t_r} - M_{g}
\]  

(23)

### 4. SIMULATION

On the basis of the presented tyre and vehicle model, which were benchmarked in a previous study [13], simulation studies were done by using the well known constant steer turn manoeuvre. It is one of the test method used in revealing the handling characteristics of a vehicle, and is also applied for the all terrain vehicles [14]. Off-road vehicle and tyre parameters were taken from the literature [12]. However, some distinctive parameters related to the simulation off-road surface type are shown in Table 1.

<table>
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<tr>
<th>Table 1. Off-road surface parameters [2]</th>
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<tr>
<td><strong>Definition</strong></td>
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<td>n</td>
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At constant longitudinal vehicle speeds, 4 degrees tyre steer angle was applied and the handling responses of the vehicle were provided for both paved and unpaved (off-road) road surface for a
comparative analysis. Simulations were done in Matlab/Simulink environment, on the basis of the developed vehicle dynamics model.

Simulations were run at various constant vehicle longitudinal velocities. Some of the selected handling responses are shown from Figure 1 to 4 for 30 and 50 km/h speeds. In the Figure 1, while the vehicle on-road almost perfectly completes its course with constant steer angle at 30 km/h, the same but off-road vehicle experiencing some amount of slipping up to 8 meters longitudinal distance. Afterwards, an oversteering tendency can be observed as it can be expected due to the loose surface properties.

![Figure 1. Provided vehicle path at 30 km/h.](image)

The similar response was provided at 50 km/h (Figure 2). However, initial slipping was increased by the increasing speed and the effective distance of the slipping was reached to about 15 meters longitudinal distance, as it is shown in Figure 3, in detail.

![Figure 2. Provided vehicle path at 50 km/h.](image)

The observed and increased slipping is due to initially lower shearing force and it perfectly depends on the type of the terrain (soil, etc). This response provided by the developed model was also declared by Bekker [6].

![Figure 3. Initiation of motion at 50 km/h.](image)

Yaw rate responses of the vehicle on road and off road are presented in Figure 4. For both velocity levels, the steady state off-road/yaw rate provided is slightly higher than the on-road/yaw rate. The result is consistent with the provided trajectories, from the point of observed oversteering tendency for the off-road vehicle.

![Figure 4. Yaw rate responses of the vehicle.](image)

5. CONCLUSIONS

In this study, a vehicle handling dynamics model with the contribution of a tyre model, which has both on-road and off-road tyre behaviour representation capabilities, was developed. The model considers a smooth sinkage and ignores arbitrary roughness. Therefore, as consistent with the yaw behaviour, roll response of the off-road is just slightly higher than the on-road. Nevertheless, longitudinal and lateral responses of the vehicle are as expected in the real world and the model proves the importance of using a proper off-road tyre model during the dynamic design stage of a terrain vehicle. The future purpose of the study...
would be introducing the terrain roughness to the model.

REFERENCES


