N- ve P-tip Katkı Mg$_{2}$Si$_{1-x}$Sn, Katı alaşımının termal iletkenliklerinin teorik çalışması

Övgü Ceyda Yelgel

ÖZ

Mg$_{2}$Si$_{1-x}$Sn, katı alaşımın yüksek termoelektrik verimlilikleri sebebiyle 500 K’den 800 K’e kadar olan orta sıcaklık aralığı için umut vaadeden termoelektrik materyallerdir. Bu çalışmada hem n- hem p-tip katkı Mg$_{2}$Si$_{1-x}$Sn, katı alaşımının termal iletkenlikleri teorik olarak detaylıca incelenmesi sunulmuştur. Taşıyıcılar (elektronlar yada holler), elektron-hole çiftlerinden ve fononlardan kaynaklanan termal iletkenlik katkıları ayrı ayrı göz önüne alınarak ve sırasıyla Wiedeman-Franz kanunu, Price’in teorisi, ve Debye’nin izotropik sürekli modeli uygulanarak hesaplanmıştır. Önegilen fonon çarpışma mekanizmaları, kaynağı kristal sınırları, Kütle bozuklukları, Deformation Potansiyelleri ve anharmoniklikten olan katı alaşımın hepsi için eksiksiz bir şekilde incelenmiştir. En düşük toplam termal iletkenlik değeri n-tip katkı Mg$_{2}$(Si$_{0.4}$Sn$_{0.6}$)$_{0.98}$Bi$_{0.02}$ katı alaşım için 700 K’da 2.431 W.K$^{-1}$.m$^{-1}$ olarak, p-tip katkı Mg$_{2}$(Si$_{0.3}$Sn$_{0.7}$)$_{0.95}$Ga$_{0.05}$ katı alaşım için 600 K’da 1.843 W.K$^{-1}$.m$^{-1}$ olarak bulunmuştur buda açıkça öneriyor ki p-tip katkı Mg$_{2}$Si$_{1-x}$Sn tabanlı katı alaşım n-tip katkı katı alaşımlarından termoelektrik cihazlar için daha iyi adaylardır.

Anahtar kelimeler: termoelektrik materyaller, termal iletkenlik, fonon termal iletkenlik

Theoretical Study of Thermal Conductivities of n- and p-type Doped Mg$_{2}$Si$_{1-x}$Sn, Thermoelectric Solid Solutions

ABSTRACT

Mg$_{2}$Si$_{1-x}$Sn, solid solutions are a promising class of thermoelectric materials due to their high thermoelectric efficiencies at intermediate temperature range from 500 K to 800 K. Present study presents a theoretical work of the thermal conductivities of both n- and p-type doped Mg$_{2}$Si$_{1-x}$Sn, solid solutions. The thermal conductivity contributions arising from carriers (electrons or holes), electron-hole pairs, and phonons are taken into account separately by employing the Wiedemann-Franz law, Price’s theory, and Debye’s isotropic continuum model, respectively. All phonon scattering mechanisms originate from crystal boundaries, mass-defects, deformation potentials, and anharmonicity are investigated rigorously for all solid solutions. The lowest total thermal conductivity values are obtained as 2.431 W.K$^{-1}$.m$^{-1}$ at 700 K for n-type doped Mg$_{2}$(Si$_{0.4}$Sn$_{0.6}$)$_{0.98}$Bi$_{0.02}$ solid solution and 1.843 W.K$^{-1}$.m$^{-1}$ at 600 K for p-type doped Mg$_{2}$(Si$_{0.3}$Sn$_{0.7}$)$_{0.95}$Ga$_{0.05}$ solid solution which clearly suggest that p-type doped Mg$_{2}$Si$_{1-x}$Sn based solid solutions are better candidates for the thermoelectric devices than their n-type doped solid solutions.

Keywords: thermoelectric materials, thermal conductivity, phonon thermal conductivity

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1. INTRODUCTION

Thermoelectric devices can transform thermal gradients to electricity and vice versa. Both thermoelectric generators and refrigerators are used for waste heat recovery. A thermoelectric device has many advantages such as they have no moving parts, have long term lifetime, are non-toxic, are noise and pollution free and are made from elements that are abundant in nature. Thermoelectric energy conversion efficiency is determined by the dimensionless thermoelectric figure of merit given by the equation as

$$ZT = \frac{S^2 \sigma}{\kappa_{total}} T,$$

where $S$ is the Seebeck coefficient (V/K), $\sigma$ is the electrical conductivity (S/m), $T$ is the absolute temperature (K), and $\kappa_{total}$ is the total thermal conductivity (W/m.K) consists three different contributions from carriers (electrons or holes; $\kappa_c$), from electron-hole pairs ($\kappa_{bp}$), and from phonons ($\kappa_{ph}$). An ideal, practical thermoelectric device should at least have a value of $ZT > 1$ which leads about 10% conversion efficiency. To achieve a higher $ZT$ has always been the motivation for the research of thermoelectric research area, however, due to strong interrelation of thermoelectric parameters $S$, $\sigma$, and $\kappa_{total}$, improving one leads to simultaneous deterioration of other two parameters. The common approach to improve $ZT$ for bulk materials is making alloys with the formation of point defects by substitutional doping. This is the most general way to improve the power factor (S2$\sigma$) by optimisation of carrier concentration and/or reduce the phonon thermal conductivity due to improving point defect scattering originates from the mass difference between host atoms and dopants [1]. One of the best thermoelectric materials are highly doped semiconductors since heat is carried predominantly by phonons thus the total thermal conductivity is largely decoupled from the power factor [1]. Among these doped semiconductors alkaline earth metal silicides Mg$_2$X ($X$=Si, Ge, and Sn) are class of materials generally used for thermoelectric device applications in the intermediate temperature range from 500 K to 800 K due to (1) relative abundance, (2) thermal stability, (3) non-toxicity, and (4) low cost of production and environment friendly [1]. These intermetallic Mg$_2$X compounds have been well known for the applications of automobiles and aerospace industry due to their high specific elastic modulus [2]. Mg$_2$X compounds are narrow band gap semiconductors and have a face-centred cubic (FCC) structure with primitive translation vectors $a = a(0, 1/2, 1/2)$, $b = a(1/2, 0, 1/2)$, and $c = a(1/2, 1/2, 0)$ where $a$ is the lattice parameter. At ambient conditions, Mg$_2$X crystallizes in cubic antifluorite type structure with space group Fm$3m$ with one molecular unit per primitive cell and four formulas per fcc conventional cell [3], as represented in Fig. 1. Sufficiently high values of $ZT$ for these Mg$_2$X ($X$=Si, Ge, and Sn) based compounds were found in the literature [4]–[7].

The $ZT$ value of undoped Mg$_2$Si was found to be $ZT = 0.04$–0.06 at nearly 750 K [8]–[10] while this could be significantly enhanced by doping with Sn, Ge, Sb, and Bi. As an example, very high value of thermoelectric figure of merit was gained for Mg$_2$Si$_{0.6}$Sn$_{0.4}$ as $ZT_{max}$=1.2 at 700 K [11] and for Mg$_2$Si$_{0.53}$Sn$_{0.47}$Ge$_{0.05}$Bi$_{0.02}$ as $ZT_{max}$=1.4 at 823 K [12]. Here thermal conductivities of n-type doped Mg$_2$(Si$_{0.4}$Sn$_{0.6}$)$_{1-y}$Bi$_y$ where $y=0.02$ and 0.03 and p-type doped Mg$_2$(Si$_{0.3}$Sn$_{0.7}$)$_{1-y}$Ga$_y$ where $y=0.05$ and 0.07 solid solutions is reported theoretically in temperature range between 300 K and 800 K. All thermal transport properties including contributions from carriers (namely donor electrons for n-type doped samples and acceptor holes for p-type doped samples) ($\kappa_c$), electron-hole pairs ($\kappa_{bp}$), and phonons ($\kappa_{ph}$) are calculated separately to compare the $\kappa_{total}$ results of both type doped samples. The Wiedemann-Franz law, Price’s theory [13], and Srivastava’s scheme [14] are employed to calculate $\kappa_c$, $\kappa_{bp}$, and $\kappa_{ph}$, respectively. All phonon scattering mechanisms including boundary scattering, mass-defect scattering,
acoustic deformation potential scattering, and anharmonic phonon scattering are taken into account rigorously. Furthermore the percentage contributions from different polarisations (longitudinal and transverse) towards the phonon thermal conductivity are presented for all types of samples.

2. THEORY

The total thermal conductivity of semiconductor materials ($\kappa_{\text{total}} = \kappa_c + \kappa_{\text{bp}} + \kappa_{\text{ph}}$) has included three contributions from carriers (electrons for n-type, holes for p-type), electron-hole pairs (bipolar), and phonons, respectively.

2.1. Carrier Thermal Conductivity

By using Wiedemann-Franz law the carrier thermal conductivity is written as [15]

$$\kappa_c = \sigma_c L_T = \left(\frac{k_B}{e}\right)^2 \sigma_c T \mathcal{L}_0, \quad (1)$$

where $k_B$ is the Boltzmann constant, $L$ is the Lorenz number and the expression of $\mathcal{L}_0$ is given by

$$\mathcal{L}_0 = \frac{(r+\frac{2}{5})F_{\frac{1}{2}}^2(\xi') - \left(\frac{r+\frac{2}{5}}{r+\frac{1}{2}}\right)^2 F_{\frac{1}{2}}^2(\xi')}{(r+\frac{2}{5})F_{\frac{1}{2}}^2(\xi')} = \left(\frac{(r+\frac{2}{5})F_{\frac{1}{2}}^2(\xi') - \left(\frac{r+\frac{2}{5}}{r+\frac{1}{2}}\right)^2 F_{\frac{1}{2}}^2(\xi')}{(r+\frac{2}{5})F_{\frac{1}{2}}^2(\xi')}, \quad (2)$$

where Fermi integral is given as $F_1 = \int_0^\infty x^4 \frac{dx}{e^{x+c}+1}$ for bulk semiconductor materials [16].

In doped semiconductors, carriers generally scatter with the long wave-length phonons which means that the most important phonon scattering mechanism is sourced from acoustic phonon scattering [17], [18]. Thus, in the extrinsic regime the electrical conductivity of semiconductors is given by [19]

$$\sigma_{\text{ext}} = \frac{4 e^2 \hbar c^2}{3\pi \sqrt{\pi} m_e^* E_D^2} F_{1/2}, \quad (3)$$

where $m_e^*$ is the carrier’s conductivity effective mass, $c_L$ is the velocity of longitudinal phonons, $\rho$ is the mass density, and $E_D$ is the deformation potential. In addition to hole acoustic phonon scattering there is also additional scattering mechanisms occur in electrical conductivity. So that, with the inclusion of these additional mechanisms the electrical conductivity can be scaled as

$$\sigma_{\text{extrinsic}} = \sigma_{\text{ext}} A T^5$$

with $A$ and $\zeta$ being adjustable parameters. Electrical conductivity in the intrinsic regime can be re-defined by using Wilson’s expression as [20]

$$\sigma_{\text{int}} = A' e^{-E_g/2k_B T}, \quad (5)$$

where $A'$ is a temperature invariant parameter. Similar to $\sigma_{\text{extrinsic}}$, due to the additional free carrier-phonon scattering in the intrinsic regime the electrical conductivity can be re-expressed as

$$\sigma_{\text{intrinsic}} = 1/\sigma_{\text{int}} + AT, \quad (6)$$

where $A'$ is treated as an adjustable parameter. The band gap $E_g$ temperature variation of n-type doped semiconductors is given as [21]

$$E_g(T) = E_g(0) - \alpha T, \quad (7)$$

and for p-type doped semiconductors is written as by following Yelgel’s formula [22]

$$E_g(T) = \left[ E_g(0) - \frac{\alpha T^2}{\beta + T} \right] + \frac{\eta T}{\beta + T'}, \quad (8)$$

where $E_g(0)$ is the value of $E_g$ at 0 K, and $\alpha$, $\beta$, and $\eta$ are treated as adjustable parameters.

2.2. Bipolar Thermal Conductivity

Mg$_2$X compounds have small band gaps in which the bipolar thermal conductivity becomes significant only above 300 K, its expression is given by [13]
where $E_g$ is a band gap of a material, $F_{bp}$ and $\zeta$ regarded as adjustable parameters.

2.3. Phonon Thermal Conductivity

Using Debye’s isotropic continuum model within the single-mode relaxation time approximation the phonon thermal conductivity is given as [14]

$$\kappa_{ph} = \frac{h^2 q_b^5}{6 \pi^2 k_B T^2} \sum_i c_i^4 \int_0^1 dx x^4 \tau_i (n + 1),$$

where $\tau$ is the phonon relaxation time, $q_D$ is the Debye radius, $x = q/q_D$ is a reduced wavenumber, $s$ represents the polarisation branch of phonon (longitudinal or transverse), $n$ is the Bose-Einstein distribution function, and $c_s$ is the velocity of phonons for polarisation branch $s$. The phonon relaxation rate ($\tau^{-1}$) is contributed by several scattering mechanisms and by using the Matthiessen rule the total effect of scattering processes is defined as $\tau^{-1} = \Sigma \tau_i^{-1}$, where $\tau_i^{-1}$ represents the contribution from $i^{th}$ scattering mechanism. The scattering mechanisms can be summarised as in the following forms.

Boundary scattering: When phonons interact with the boundaries of a sample the scattering mechanism is defined as

$$\tau_{qs}^{-1}(bs) = \frac{c_s}{L},$$

where $L$ is a crystal size [14], [23].

Mass defect scattering: The scattering of phonons from isotopes and alloying in semiconductor alloys and compounds is written as [14], [24]

$$\tau_{qs}^{-1} = \frac{\Gamma_{nd} \Omega}{4 \pi c^3} w^4 (qs),$$

where $\Omega$ is the volume of a unit cell, $c$ is the average phonon velocity, $w = c q$ and $\Gamma_{nd}$ is the mass-defect parameter. For a single-species crystal, the isotopic mass-defect parameter takes the form [24]

$$\Gamma_{isotopes} = \sum_i f_i \left( \frac{\Delta M_i}{M} \right)^2,$$

where $f_i$ is the percentage of $i^{th}$ isotope present in the crystal and $\Delta M_i = M_i - \bar{M}$, with $M$ is the average atomic mass. For a composite material, such as an alloy, with molecular formula $A_x B_y C_z$... the mass-defect parameter is given by [24]

$$\Gamma_{allo} (A_x B_y C_z ... ) = \frac{x}{(x+y+z+...) \frac{M_A}{M}} \frac{M_A}{M} \Gamma (A) + \frac{y}{(x+y+z+...) \frac{M_B}{M}} \frac{M_B}{M} \Gamma (B) + \frac{z}{(x+y+z+...) \frac{M_C}{M}} \frac{M_C}{M} \Gamma (C) + ..., (14)$$

where $\Gamma (A) = \sum_i f_i \left( \frac{\Delta M_i (A)}{M} \right)^2$ represents the defect parameter for atomic species $A$, and the average atomic mass is expressed as $\bar{M} = (x M_A + y M_B + z M_C + ...)/(x + y + z + ...)$.

Acoustic deformation potential scattering: The scattering of charge carriers (electrons for n-type samples, holes for p-type samples) with acoustic mode phonons is expressed as [25]

$$\tau_{q}^{-1} (dp) = \frac{3}{8 \pi^2 \rho c^5 k_B T^{3/2}},$$

where $E_{df}$ is a scaled deformation potential.

Anharmonic phonon scattering: By restricting ourselves to only three phonon processes and following Srivastava’s scheme [14] the phonon-phonon scattering mechanism is given by

$$\tau_{q}^{-1} (an) = \frac{h q_b^2}{4 \pi^2 \rho c^5} \sum_{x',x''} \int dx^' x^' x^'' (1 - \varepsilon + \varepsilon(\varepsilon + 2)) \left[ \int dx x x^' x^'' (1 - \varepsilon + \varepsilon(\varepsilon + 2)) \right] \left[ \frac{n_{q}x'}{n_{q}x} \right] \left[ \frac{n_{q}x''}{n_{q}x'} \right] \left[ \frac{n_{q}x'}{n_{q}x''} \right],$$

where $\gamma$ is the Grüneisen constant, $x' = q' q_d$, $x'' = n(x'')$, $C = c_s/c_d$, $D = c_s/c_c$, $\varepsilon = 1$ for momentum-conserving processes and $\varepsilon = -1$ for momentum-nonconserving processes. The first and second terms in Eq. (17) are controlled by class 1 events $(q s + q s' \rightarrow q s')$ and class 2 events $(q s \rightarrow q s' + q s')$, respectively. The integration limits on the variables $x$ and $x'$ have already been given in Ref. [14].

3. RESULTS AND DISCUSSION

Thermal conductivities are performed for n-type doped Mg$_2$(Si$_{0.4}$Sn$_{0.6}$)$_{1-y}$Bi$_y$ ($y=0.02$ and $0.03$) and p-type doped Mg$_2$(Si$_{0.3}$Sn$_{0.7}$)$_{1-y}$Ga$_y$ ($y=0.05$ and $0.07$) solid solutions in the temperature range $300 \text{ K} \leq T \leq 800 \text{ K}$. All the required properties and parameters are listed in Tab. 1. The carrier thermal conductivities of both type doped solid solutions are calculated and represented in Fig. 2.
For both n- and p-type doped samples $\kappa_c$ goes up with higher doping level because of rise in the electrical conductivity as the doping level increases. The reason of significantly lower $\kappa_c$ values for p-type doped samples originates from their lower electrical conductivities both with their smaller Lorenz numbers and band gap values compared to n-type doped samples. Additionally, from our theoretical calculations it is found that for both n- and p-type doped samples throughout the temperature range $\kappa_c$ increases with temperature smoothly. The theoretical calculation of the sum of phonon and bipolar thermal conductivities for both type doped samples are shown in Fig. 3. From our theoretical calculations it is found that for both type doped samples boundary and carrier-phonon scatterings are important at temperatures below 100 K, the anharmonic phonon scattering is important at temperatures above 100 K and mass-defect scatterings become significant both at low and high temperatures. Additionally, it is found theoretically that for both type samples $\kappa_{bp}$ increases exponentially with temperature and it becomes significant for n-type doped samples above 550 K while for p-type doped samples above 650 K. The increment in $\kappa_{bp}$ from 500 K to 800 K is faster for n-type doped samples than p-type doped samples. Moreover, from our theoretical calculations the smaller values of $\kappa_{bp}$ is gained for p-type doped samples due to their smaller energy band gaps at. Finally, as shown in Fig. 3 n-type doped samples have lower $\kappa_{ph} + \kappa_{bp}$ values than p-type doped samples the reason for this is even n-type doped samples have higher $\kappa_{bp}$ values they have lower $\kappa_{ph}$ values due to their larger mass-defect scatterings defined by $\Gamma_{md}$ parameters as listed in Tab. The temperature variation of the percentage contribution from different polarisations of phonon thermal conductivity for both n- and p-type doped samples is presented in Fig. 4 and Fig. 5, respectively.
with 51% for p-type doped Mg2(Si0.3Sn0.7)1−yGa0.05 and y=0.05 solid solutions and p-type doped Mg2(Si0.3Sn0.7)1−yGa0.07 with y=0.03 solid solutions, respectively.

The critical temperature (Tc) decreases with temperature and after reaching at 600 K this heat is mostly carried by transverse phonons with nearly 56% at 300 K this normal conductivities of both type doped samples are demonstrated in Fig. 6. From the theoretical calculations the lower values of κtotal is gained for p-type doped samples due to their significantly smaller carrier thermal conductivities whereas they have lower phonon thermal conductivities than n-type doped samples. The lowest value of κtotal is found to be for p-type doped Mg2(Si0.3Sn0.7)0.93Ga0.07 solid solution as 1.843 W K−1 m−1 at 600 K whereas the lowest κtotal value for n-type doped sample is gained for n-type doped Mg2(Si0.3Sn0.6)0.97Bi0.03 solid solution as 2.431 W K−1 m−1 at 700 K. These findings clearly suggest that by using p-type doped Mg2Si1−ySnx based solid solutions instead of their n-type doped types are likely to enhance the value of thermoelectric figure of merit.

4. CONCLUSION

The theoretical study of the three different thermal conductivity contributions (sourced from carriers (electrons or holes), electron-hole pairs, and phonons) for n-type doped Mg2(Si0.3Sn0.6)1−yBi0.05 with y=0.02 and 0.03 and p-type doped Mg2(Si0.3Sn0.7)1−yGa0.05 with y=0.05 and 0.07 solid solutions is presented in the temperature range 300 K ≤ T ≤ 800 K. The following conclusions can be made from our theoretical investigations:

1. The lowest carrier thermal conductivity are found to be 1.227 W K−1 m−1 at 300 K for n-type doped Mg2(Si0.3Sn0.6)0.98Bi0.02 sample and 0.451 W K−1 m−1 at 300 K for p-type doped Mg2(Si0.3Sn0.7)0.95Ga0.05 sample.

2. The smallest values of κph + κsp are gained as 0.879 W K−1 m−1 at 600 K for n-type doped Mg2(Si0.3Sn0.6)0.97Bi0.03 sample and 1.277 W K−1 m−1 at 600 K for p-type doped Mg2(Si0.3Sn0.7)0.95Ga0.05 sample.
(3) For an efficient thermoelectric material one needs to have smaller $k_{\text{total}}$ value. In this theoretical study significantly lower total thermal conductivity values are gained for both n- and p-type doped Mg$_2$Si$_{1-x}$Sn$_x$ based solid solutions. The lowest $k_{\text{total}}$ values are obtained as 2.431 $\text{WK}^{-1}\text{m}^{-1}$ at 700 K for n-type doped Mg$_2$(Si$_{0.4}$Sn$_{0.6}$)$_{0.96}$Bi$_{0.02}$ sample and 1.843 $\text{WK}^{-1}\text{m}^{-1}$ at 600 K for p-type doped Mg$_2$(Si$_{0.3}$Sn$_{0.7}$)$_{0.95}$Ga$_{0.05}$ sample. This result suggests that p-type doped Mg$_2$Si$_{1-x}$Sn$_x$ based solid solutions are better candidates for the thermoelectric devices than their n-type doped solid solutions.

Table 1: Related parameters employed in the computation of thermoelectric properties of n-type doped Mg$_2$(Si$_{0.4}$Sn$_{0.6}$)$_{1-y}$Bi$_y$ where $y=0.02$ and 0.03 and p-type doped Mg$_2$(Si$_{0.3}$Sn$_{0.7}$)$_{1-y}$Ga$_y$ where $y=0.05$ and 0.07 solid solutions.

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