Design of discrete time controllers for the DC-DC boost converter

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ABSTRACT

In this paper, a small signal model for the boost converter is obtained. Two digital controllers using pole placement technique and Linear Quadratic Optimal Regulator (LQR) methods are designed and applied to the boost converter. The compensated system’s operations and analysis are discussed and verified through MATLAB/Simulink simulation. Comparison between the two controllers related to the design methodology, implementation issues and transient measured performance is carried out.

Keywords: Boost Converter, Small Signal Model, pole placement, LQR.

Yükseltici tip DC-DC dönüşümler için ayrık-zaman kontrolör tasarımını

ÖZ


Anahtar kelimeler: Yükseltici tip DC-DC dönüşümler, Küçük-İşaret Modeli, Kutup Yerleştirme, LQR.

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1. INTRODUCTION

DC-DC converters are extensively used in distributed power supply systems and modern power electronics devices due to their high efficiency, high power density, high power levels, low cost, and small size [1]. They are also used extensively in uninterruptible power supply, power factor improvement, harmonic elimination, fuel cells applications and in photovoltaic arrays [2]. In addition, they can be step-up or step-down converters, and can have multiple output voltages. A wide variety of topologies is employed by switched-mode DC-DC converters technology [3].

Boost converter, (also known as a step-up converter) is a type of switched-mode DC-DC converter which produces a constant output voltage that is greater than input voltage. Averaging techniques such as small signal model has been widely used to derive model of DC-DC converters [4]-[6]. This circuit model is used to simulate the system and design the suitable controller. But, the small signal model changes related to the variations in the operating points [7].

PID controller is a traditional linear control method used in many applications. Linear PID controllers for DC-DC converters are usually designed by frequency domain methods applied to the small signal models of the converters. PID controller response could be poor against changed in the operating points [7]-[9].

Frequency domain methods of design such as root locus techniques or frequency response techniques can’t design and specify all closed loop poles of the higher order system since those methods don’t have sufficient parameters to place all of the closed loop poles. State space methods such as state feedback controller solve this problem by introducing into the system other adjustable parameters [10].

State feedback control and the approach of linear quadratic optimal regulator (LQR) have a good control solution for the systems with good dynamic response, accepted robustness, output regulation, and disturbances rejection.

This paper is organized as follows; small signal state space modeling of boost converter is carried out. Later an integral controller and a state feedback controller based on pole placement technique and LQR methods are briefly discussed and applied to the boost converter system. Finally, simulation and experimental results are obtained to verify the performance responses under different operating points and input variations.

2. MODELING OF BOOST CONVERTER

Basic circuit for the boost converter is shown in Fig. 1. Circuit consists of an inductor (L), a power switching element (S), a diode, a filter capacitor (C), and a load resistor (R). The switch S is turned on and off at switching frequency \( f_s = \frac{1}{T_s} \), where \( T_s \) is the switch cycle period. Duty cycle \( d \) is the ratio of the on-time interval to the switch cycle period \( d = \frac{t_{on}}{T_s} \).

![Figure 1. Basic circuit of Boost Converter](image)

where, \( i_L(t) \) is the inductor current and \( v_o(t) \) is the output voltage which is equal to the capacitor voltage. As shown in Fig. 1, the circuit is driven using a pulse train comes from the control signal \( u \). The dynamic equations of the boost converter are obtained during one switching cycle as follow:

\[
\begin{align*}
(0 < t < dT_s), \quad u & = 1 \\
& \left\{ \begin{aligned}
\frac{dl}{dt} &= \frac{1}{L} V_{in} \\
\frac{dv_o}{dt} &= -\frac{1}{RC} V_o
\end{aligned} \right. \\
(\text{dT}_s < t < T_s), \quad u & = 0 \\
& \left\{ \begin{aligned}
\frac{dl}{dt} &= \frac{1}{L} V_{in} - \frac{1}{L} V_o \\
\frac{dv_o}{dt} &= \frac{1}{C} i_L - \frac{1}{RC} V_o
\end{aligned} \right.
\]

By combining the above two equations in one general equation, then:

\[
\left\{ \begin{aligned}
\frac{dl}{dt} &= \frac{1}{L} V_{in} - (1 - u) \frac{1}{L} V_o \\
\frac{dv_o}{dt} &= -\frac{1}{RC} V_o + (1 - u) \frac{1}{C} i_L
\end{aligned} \right. \tag{1c}
\]

A number of methods are appeared for ac equivalent circuit modeling such as circuit averaging, current injected approach, averaged switch modeling and state space averaging method [11]. The state space averaged modeling is widely used in DC-DC Converter’s modeling since it achieves a certain performance objective and provides an accurate model [7], [25]. The state space averaged model can be obtained by averaging the state space differential equations which illustrated in equation (1) using the duty ratio. It is considered that state and control variables have small dc variations/disturbances around the steady state operating...
point. Therefore, every parameter has a steady state (dc) term and a dynamic (ac) term. Small signal model is used for control purposes and controller design, and it is obtained by eliminating the dc term [7], [12].

2.1. Boost Converter’s Small-Signal Model

Small signal model gives an idea of the dynamics and variations about steady state operating point and it is represented by (>). The small signal model of the boost converter which is obtained using state space averaging techniques is [12]:

$$\frac{d}{dt} \begin{bmatrix} \hat{i}_L \\ \hat{\varnothing}_o \end{bmatrix} = \begin{bmatrix} A & B \\ C & \frac{1}{1+\frac{L}{RC}} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{\varnothing}_o \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \hat{v}_{in} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \hat{d} \quad (2a)$$

$$\hat{\varnothing}_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{\varnothing}_o \end{bmatrix} \quad (2b)$$

In equation (2), \(\hat{i}_L\) and \(\hat{\varnothing}_o\) is the small signal ac variation of the inductor current and the output voltage respectively which are considered as the state variables, \(i_L\) can considered as the nominal inductor current, and \(D\) is the nominal duty cycle which is given by:

$$D = 1 - \frac{\hat{v}_{in}}{v_o} \quad (3)$$

Boost converter system has one output (output voltage \(\hat{v}_o\)), one control input (duty cycle \(\hat{d}\)), and some external inputs as (the load current \(\hat{i}_L\) or variations in the input voltage \(v_{in}\)) could be considered to the system as disturbance inputs since they can be changed related to the application. Then from equation (2), since the duty cycle \(\hat{d}\) is taken as the control input, the small signal model is considered to be like [18]-[20]:

$$\frac{d}{dt} \begin{bmatrix} \hat{i}_L \\ \hat{\varnothing}_o \end{bmatrix} = \begin{bmatrix} A & B \\ C & \frac{1}{1+\frac{L}{RC}} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{\varnothing}_o \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \hat{v}_{in} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \hat{d} \quad (4a)$$

$$\hat{\varnothing}_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{\varnothing}_o \end{bmatrix} \quad (4b)$$

As the digital controller will be used in the boost converter application system, the design analysis will be considered from the concepts of a discrete-time system. Then equation (4) should be discretized as follow:

$$x[k + 1] = Gx[k] + Hu[k] \quad (5)$$

where, \(G\) and \(H\) depend on the sampling time \(T\), and it can be derived as follow [14]:

$$G = e^{AT}, \quad H = \left( \int_0^T e^{AT} d\tau \right) B \quad \text{and} \quad C_d = C \quad (6)$$

Table 1: Design values of the Boost Converter

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameters</th>
<th>Design Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Input Voltage, (V_{in})</td>
<td>24V</td>
</tr>
<tr>
<td>2.</td>
<td>Output Voltage, (V_o)</td>
<td>50V</td>
</tr>
<tr>
<td>3.</td>
<td>Inductance, (L)</td>
<td>72(\mu)H</td>
</tr>
<tr>
<td>4.</td>
<td>Capacitance, (C)</td>
<td>50(\mu)F</td>
</tr>
<tr>
<td>5.</td>
<td>Load Resistance, (R)</td>
<td>23(\Omega)</td>
</tr>
<tr>
<td>6.</td>
<td>Switching Frequency, (f_s)</td>
<td>100KHz</td>
</tr>
</tbody>
</table>

3. CONTROLLER DESIGN

Circuit parameters for the prototype boost converter are listed in table 1. By using these values, a small signal state space model can be obtained as constructed in equation (7). The sampling frequency is assumed to be 100 KHz (sampling time \(T = 10\mu\)s). The designed boost converter has two complex stable poles at 0.992 \(\pm j0.0795\) and one unstable zero at 2.17 as shown in Fig. 2. The unstable zero makes limitations on the obtainable closed loop bandwidth of the controlled converter. These limitations come from a fundamental nature and not causes from a particular design criterion [15].

$$\begin{bmatrix} \hat{i}_L[k + 1] \\ \hat{\varnothing}_o[k + 1] \end{bmatrix} = \begin{bmatrix} 0.9968 & -0.0663 \\ 0.9955 & 0.9882 \end{bmatrix} \begin{bmatrix} \hat{i}_L[k] \\ \hat{\varnothing}_o[k] \end{bmatrix} + \begin{bmatrix} 6.9671 \\ -0.5687 \end{bmatrix} \hat{d} \quad (7a)$$

$$\hat{\varnothing}_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{i}_L[k] \\ \hat{\varnothing}_o[k] \end{bmatrix} \quad (7b)$$

This open loop state space of the prototype boost converter system will be considered in the next digital controller design. Digital controller will be designed and obtained with two control techniques.

Figure 2. Boost Converter's pole and zero locations in discrete-time domain
3.1. State Space Technique Method

A state space is a method for designing, modeling and analyzing a feedback control systems which can be applied to a wider class of systems. In frequency domain methods such as root locus technique, just one gain adjustment or a compensator selection is not enough to produce sufficient parameters to place all the closed loop poles at the desired locations. Designers need n-adjustable parameters to place n-quantities [10].

![Schematic diagram of a full-state feedback system](image)

Figure 3. (a) Plant with state feedback, (b) Basic state-feedback with integral actuation

State space methods such as pole placement and Linear Quadratic Optimal Regulator (LQR) introduce into the system another adjustable parameters and the method to find these parameters values, thus all closed loop poles can be placed at the desired locations [10].

3.1.1. Controller Design Via Pole Placement Technique

Fig. 3(a) illustrates the schematic diagram of a full-state feedback system. The topology in state feedback design is to take a feedback path from every state variable to the control input “u” through a gain “K” instead of taking a feedback path from the output “y” as shown in Fig. 3(a). Therefore, there are n-gains will be adjusted to meet the desired closed loop pole locations which obtain the required transient response. The main necessary condition for pole placement is that the system should be completely state controllable [2]. The control input “u” will be like:

\[ u[k] = r[k] - Kx[k] \]  

The state equations for the closed loop system of Fig. 3(a) can be represented by:

\[ x[k + 1] = Gx[k] + Hu[k] \]  

\[ y[k] = C_d x[k] \]  

The previous step is designed to meet the transient requirements, and then the design of steady state error characteristic should be addressed since an error signal will be introduced to the closed loop system [2] [8] [14]. A feedback path from the output “y” is added to be compared with a reference input. And then an error “e” is identified, which is fed forward to the controlled plant though an integrator as shown in Fig. 3(b). The integrator increases the system’s type and eliminates the steady state error. Therefore, additional state variable “v” has been added at the output of the integrator, then:

\[ v[k] = v[k - 1] + e[k] \]  

\[ v[k] = v[k] + r[k] - C_d x[k] \]  

Equation (10) can be re-written as:

\[ v[k + 1] = v[k] + r[k + 1] - C_d x[k + 1] \]  

\[ v[k + 1] = v[k] + r[k + 1] - C_d (Gx[k] + Hu[k]) \]

\[ = v[k] + r[k + 1] - C_d Gx[k] - C_d Hu[k] \]  

where v[k] is the actuating error vector, and r[k] is the command input vector. Therefore, the updated state space will be expressed like [13]:

\[ \begin{bmatrix} x[k + 1] \\ v[k + 1] \end{bmatrix} = \begin{bmatrix} C_d & 0 \\ -C_d G & 1 \end{bmatrix} \begin{bmatrix} x[k] \\ v[k] \end{bmatrix} + \begin{bmatrix} H \\ -C_d H \end{bmatrix} u[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r[k + 1] \]  

\[ y[k] = [C_d & 0] \begin{bmatrix} x[k] \\ v[k] \end{bmatrix} \]  

The control vector u[k] is given by:

\[ u[k] = -Kx[k] + k_v v[k] = -[K - k_v] \begin{bmatrix} x[k] \\ v[k] \end{bmatrix} \]  

By substituting equation (13) into equation (12), then the closed loop state is given by:

\[ \begin{bmatrix} x[k + 1] \\ v[k + 1] \end{bmatrix} = \begin{bmatrix} A_d \\ 0 \end{bmatrix} \begin{bmatrix} (G - HK) & H k_v \\ -C_d G + C_d H k_v & 1 - C_d H k_v \end{bmatrix} \begin{bmatrix} x[k] \\ v[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r[k] \]  

\[ \begin{bmatrix} A_d \\ 0 \end{bmatrix} \begin{bmatrix} (G - HK) & H k_v \\ -C_d G + C_d H k_v & 1 - C_d H k_v \end{bmatrix} \]
\[ y[k] = [G_d \ 0] \begin{bmatrix} x[k] \\ v[k] \end{bmatrix} \]  

(14b)

where \( K = [k_1 \ k_2] \) since the system contains two state variables, and \( A_d \) will be considered as the system matrix of the state space in equation (14). The desired closed loop locations will be decided, and then an equivalent characteristic equation will be determined. By equating the same coefficients of characteristic equation associated with equation (14), \( K \) and \( K_i \) can be solved to meet the desired transient response. In this study, the characteristic equation to determine the unknown values of \( K \) and \( K_i \) is formed as follow:

\[
|zI - A_d| =
\begin{vmatrix}
z^2 + (-2.985 + 6.9671 k_1 - 0.568 k_2 - 0.5687 k_i) z^2 + (2.9764 - 13.8143 k_1 + 1.8 k_2 + 1.2322 k_i) z + (6.8472 k_1 - 1.2322 k_2 - 0.9914) & = 0
\end{vmatrix}
\]

(15)

The desired discrete characteristic equation formed by two dominant poles with a damping ratio \( \zeta = 0.95 \) and a settling time \( t_s = 1 ms \), and a third real pole which is near to the unstable zero and has a real part many times greater than the desired dominant second order poles in order to not affect the transient response.

\[
P(z) = (z - 0.9607) (z - 0.3679)
\]

(16)

By equating the same coefficients of \( z \) in equations (15) and (16), the state feedback matrices are obtained as follow:

\[
K_i = 172 T = 0.00172 \\
K = [k_1 \ k_2] = [0.104 \ 0.049]
\]

(17)

### 3.1.2. Controller Design Via LQR Technique

State feedback techniques to locate the closed loop poles at any needed position. Pole placement technique is not so much robust against the external disturbances which could be introduced to the system resulting modeling error. Therefore, pole placement techniques could lead to unsatisfactory results in system performance \([16], [21]\). Linear Quadratic Optimal Regulator (LQR) is a method to choose state feedback gain “\( K \)”, and it is still a powerful tool in term of tuning the state feedback gain “\( K \)” to obtain the optimal control law given by \( u[k] = -Kx[k] \).

System must be observable and also state controllable \([2] [14] [17]\). Then, LQR obtains an optimal response related to the designer’s specifications in such a way that the dominant closed loop poles are assigned close to the desired locations and the remaining poles are non-dominant. This method assures insensitivity to plant parameter variations by choosing appropriate performance index. On other words, states or outputs of the control system are kept within an acceptable deviation from a reference condition using acceptable expenditure of control effort \([22]-[24]\). In addition, it can be applied with the independence of system’s order and can be easily calculated from the matrices of the system’s small signal model \([14]\). The idea of LQR method is to minimize the performance index as follow \([14]\):

\[
J = \frac{1}{2} \sum_{k=1}^{N} [X^T[k] Q X[k] + U^T[k] R U[k]]
\]

(18)

where \( X \) and \( U \) are the state and control vectors respectively, and \( Q \) and \( R \) are real and positive definite symmetric constant matrices. These \( Q \) and \( R \) are considered as weighting matrices. It is required to have \( Q \) and \( R \) matrices with a some number that drive the states to zero during the summation process, thus minimize the cost function “\( J \)”. Therefore, by choosing the values of \( Q \) and \( R \), the relative weighting of one state versus another can be changed and adjusted. Based on equation (13) and Fig. 3(b) which illustrates the basic schematic diagram of the state feedback system, the control vector \( u[k] \) will be considered as follow:

\[
u[k] = -Kx[k] + k_1 v[k]
\]

(19)

It is assumed that the \( K_d \) gain matrix which is the solution of the above closed loop control law is determined and solved optimally and it is given by:

\[
K_d = (H_d^T P G_d + R)^{-1} H_d^T P G_d
\]

(20)

Where, \( G_d \) and \( H_d \) is the system’s matrices as considered in equation (12). \( P \) is determined to satisfy the following reduced discrete-time Riccati matrix equation given by:

\[
G_d^T P G_d - P - C_d^T P H_d (H_d^T P H_d + R)^{-1} H_d^T P G_d + Q = 0
\]

(21)

For the appropriate “\( P \)” value, system should be asymptotically stable in the steady state condition \([2] [14]\). By substituting of the “\( P \)” value in the equation described in equation (20), the value of optimal “\( K_d \)” gain matrix can be obtained. In this study, and after some trial and error and numerous simulations, the positive definite \( Q \) and \( R \) for this converter is assumed as:

\[
Q = \begin{bmatrix}
100 & 0 & 0 \\
0 & 1000 & 0 \\
0 & 0 & 1.7
\end{bmatrix}
\]

and \( R = 1 \)

(22)
By solving the LQR problem for $K_e$, the $K_e$ values of this converter is obtained like follows:

$$K_e = 1500T = 0.015$$
$$K = [0.2157 \quad 0.3942]$$

(23)

4. SIMULATION & RESULTS

The design and the performance of boost converter is accomplished in continuous conduction mode with circuit parameters as tabulated before in Table 1. Simulations of boost converter with the previous discussed two controllers are obtained using MATLAB/Simulink.

![Simulink Model of the controlled Boost Converter](image)

The performance parameters of the boost converter under consideration: rise time, settling time, maximum peak overshoot and steady state error are simulated and tabulated in the Table 2 with the two controller cases. In addition, the performance of the state feedback controlled boost converter with every controller case is checked under tracking the reference voltage as shown in Fig. 5. Also, it is checked under the effects of sudden changes in the input voltage and load as shown in figures 6 and 7 respectively. The nominal input voltage and reference voltage for the boost converter are adjusted to 24V and 50V respectively, where the nominal load is 23Ω as considered before in Table 1.

The first test is performed by changing the reference voltage in this sequence: 50V, 60V and 40V respectively with fixed input voltage and load at their nominal values. The output response of the boost converter in the two controller cases tracks the reference voltage with no steady state error and a fixed output voltage regulation as shown in Fig. 5.

![Simulation Results for the Controlled Boost Converter’s output response under reference voltage variations](image)

The second test is performed by changing the input voltage in this sequence: 24V, 12V, and 35V respectively with fixed load at the nominal value. The output voltage response of the boost converter in the two controller cases shows fixed output voltage regulation irrespective of the input voltage variations as shown in Fig. 6.

![Simulation Results for the Controlled Boost Converter’s output response under input voltage variations](image)
The third test is performed by changing the load in this sequence: 23Ω, 28Ω and 20Ω respectively with fixed input voltage at the nominal value. The output response of the boost converter in the two controller cases tracks the reference voltage with no steady state error and with a fixed output voltage regulation irrespective of the load variations as shown in Fig. 7.

![Figure 7](image)

Figure 7. Simulation Results for the Controlled Boost Converter’s output response under load variations. (a) LQR Method. (b) Pole Placement Method. (c) Load

5. CONCLUSION

In this paper, two digital controllers based on pole placement technique and Linear Quadratic Optimal Regulator (LQR) methods have been designed and implemented using MATLAB/Simulink for DC-DC boost Converter. The designed controllers provide excellent static and dynamic characteristics at all operating points. As shown in this paper, the Linear Quadratic Optimal Regulator (LQR) based controller shows very good output voltage regulation, excellent dynamic performances and higher efficiency irrespective of the operating point’s variations and the external disturbances.

6. REFERENCES

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